

Nonequilibrium dynamics of a spin-3/2 Blume Capel model with quenched random crystal field

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The relaxation and complex magnetic susceptibility treatments of a spin-3/2 Blume-Capel model with quenched random crystal field on a two dimensional square lattice are investigated by a method combining the statistical equilibrium theory and the thermodynamics of linear irreversible processes. Generalized force and flux are defined in irreversible thermodynamics limit. The kinetic equation for the magnetization is obtained by using linear response theory. Temperature and also crystal field dependencies of the relaxation time are obtained in the vicinity of phase transition points. We found that the relaxation time exhibits divergent treatment near the order-disorder phase transition point as well as near the isolated critical point whereas it displays cusp behavior near the first order phase transition point. In addition, much effort has been devoted to investigation of complex magnetic susceptibility response of the system to changing applied field frequencies and it is observed that the considered disordered magnetic system exhibits unusual and interesting behaviors. Furthermore, dynamical mean field critical exponents for the relaxation time and complex magnetic susceptibility are calculated in order to formulate the critical behavior of the system. Finally, a comparison of our observations with those of recently published studies is represented and it is shown that there exists a qualitatively good agreement.

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I. INTRODUCTION

Investigation of disorder effects on the critical phenomena has a long history and there have been a great many of theoretical studies focused on disordered magnetic materials with quenched randomness where the random variables of a magnetic system such as random fields [1, 2] or random bonds [3, 4] may not change its value over time. On the other hand, quenched crystal field diluted ferromagnets constitute another example of magnetic systems and equilibrium properties of this type models have been investigated under multifarious approximation methods [5–8]. Besides, effects of disorder on magnetic systems have been systematically studied, not only for theoretical interests but also for the identifications with experimental realizations [9–11]. It has been shown by renormalization group arguments that first-order transitions are replaced by continuous transitions, consequently tricritical points and critical end points are depressed in temperature, and a finite amount of disorder will suppress them [12].

Since the time in which Ising model [13] was invented there exists a limited number of studies including the dynamic nature of the system under the influence a small perturbation. Even though the physical investigations regarding these systems bring about a lot of mathematical difficulties, the nonequilibrium systems are in the focus of scientists because they have an unusual and interesting dynamic behavior. It is known that examina-

tion of physical properties of the nonequilibrium systems allows us to analyze various dynamical concepts. One of them is the relaxation time of a considered system which exhibits a divergence near the critical and multicritical points and gives remarkable information about dynamic properties. As far as we know, on the theoretical picture, the relaxation behavior of a limited number of systems has been investigated by making use of Onsager reciprocity theorem [14, 15]. This type of investigation was first performed by Tanaka *et al.* [16] near the continuous phase transition point for the spin-1/2 Ising model by means of Onsager reciprocity theorem. Next, AB-type ferromagnetic and antiferromagnetic models have been investigated by Barry [17], Barry and Harrington [18] using the similar formulation, respectively. Additionally, in Ref. [18], dynamic initial parallel susceptibility expression is analyzed near the Néel temperature for considered applied field frequencies. Recently, spin-1 Ising model has been analyzed by Erdem and Keskin within the framework of the entropy production [19, 20] and it is found that one of the relaxation times diverges at the continuous phase transition point while the other relaxation time remains finite. The relaxation process of the pure spin-3/2 Blume-Emery-Griffiths (BEG) model with bilinear and biquadratic exchange interactions has been analyzed near the critical [21] and multicritical points [22]. In addition, thermal variations of the relaxation time of a metamagnetic Ising model has been investigated by Gulpinar *et al.* [23]. Very recently, following the same methodology, spin-1 Blume-Capel (BC) model with quenched random crystal field has been studied in detail and it is observed that the relaxation time has a jump discontinuity at the first-order phase transition

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point whereas it diverges in the neighborhood of multicritical points such as tricritical, critical end point and bicritical end points [24]. Moreover, from the experimental point of view, a great many of experimental studies have been dedicated to the better understanding of dynamic critical phenomena in magnetic systems [25–29].

On the other side, it is known that the dynamic or ac susceptibility measurement, in which a time varying oscillating magnetic field is carried out to a sample, is a powerful method for analyzing the dynamic evolution of the considered real magnetic system. It is obtained from the dynamic response of the system to time dependent magnetic field and many studies have been performed regarding the magnetic relaxation of cooperatively interacting systems, such as nanoparticles [30–32], spin glasses [33], high T_c systems [34] and magnetic fluids [35]. From the theoretical point of view, in Ref. [36], Acharyya and Chakrabarti probed thoroughly the real and imaginary parts of magnetic susceptibility in the neighborhood of phase transition temperature of a spin-1/2 Ising model under the time dependent oscillating external magnetic field by using Monte Carlo (MC) simulation with periodic boundary conditions. Additionally, Erdem investigated the magnetic relaxation in a spin-1 Ising model near the second-order phase transition point where time derivatives of the dipolar and quadrupolar order parameters are treated as fluxes conjugate to their appropriate generalized forces in the sense of irreversible thermodynamics [37], next, the same author has analyzed the frequency dependence of the complex susceptibility for the same system [38]. In addition to these, with the helping of Nelson's method, a systematic investigation containing frequency, momentum and temperature dependent response function has been achieved for Ising-type system below the critical temperature [39]. Very recently, a comprehensive study including the equilibrium and nonequilibrium antiferromagnetic and ferromagnetic susceptibilities of a metamagnetic Ising system in the vicinity of order-disorder transition point has been done by benefiting from mean field approximation (MFA) [40].

Apart from these, the effects of impurities on driving-rate-dependent energy loss in a ferromagnet under the time dependent magnetic field have been analyzed by Zheng and Li [41] by using several well defined models within the frameworks of MFA and MC, and they found using MFA that, the hysteresis loop area is a power law function of the linear driving rate as $A - A_0 \propto h^\beta$, where, A_0 , h and β are the static hysteresis loop area, the linear driving rate and scaling exponent of the system, respectively. Very recently, the quenched site and bond diluted kinetic Ising models under the influence of a time dependent oscillating magnetic field have been analyzed by making use of effective field theory [42, 43] on a two dimensional honeycomb lattice. In Ref. [42], the global phase diagrams including the reentrant phase transitions are presented by the authors for site diluted kinetic Ising model, and they showed that the coexistence regions disappear for sufficiently weak dilution of

lattice sites. Following the same methodology, the authors have concentrated on the influences of quenched bond dilution process on the dynamic behavior of the system. After some detailed analysis, it has been found that the impurities in a bond diluted kinetic Ising model give rise to a number of interesting and unusual phenomena such as reentrant phenomena and the impurities to have a tendency to destruct the first-order transitions and the dynamic tricritical point [43]. Furthermore, it has also been shown that dynamically ordered phase regions get expanded with decreasing amplitude which is more evident at low frequencies.

As far as we know, the nonequilibrium properties of the quenched disordered systems such as crystal field (and also bond or site) diluted ferromagnets under the influence of a small magnetic field perturbation have not yet been investigated for higher spin models. These types of disorder effects constitute an important role in the real magnetic material science, since the quenched disorder effects may induce some important macroscopic influences on the material. Therefore, we believe that the investigation of the effects of quenched randomness on the nonequilibrium properties of the Ising model and its derivations still need particular attention. Hence, in this work based on MFA, we intend to study the nonequilibrium properties of a spin-3/2 BC model by introducing the quenched crystal field dilution effects. Here, we used a method combining statistical equilibrium theory and the thermodynamics of irreversible processes to analyze the magnetic relaxation behavior and dynamic complex susceptibility of a spin-3/2 BC model with quenched crystal field randomness. In this context, by means of the MFA to obtain magnetic Gibbs free-energy, a generalized force and a generalized current are determined within the framework of irreversible thermodynamics in the neighborhood of equilibrium states. After that, the kinetic equation for the time dependent magnetization is obtained and a relaxation time is derived and also thermal and crystal field variations are examined in the neighborhood of critical and isolated critical points. In addition, we calculated the complex magnetic susceptibility expression and analyzed near the isolated critical as well as second order phase transition points for selected Hamiltonian parameters. It can be said that the considered disordered magnetic system exhibits unusual and interesting behaviors. The aforementioned features will be discussed in detail in later sections.

The remaining part of the paper is as follows: In Sec. II we briefly present the formulations, the results and discussions are presented in Sec. III, and finally Sec. IV contains our conclusions.

II. FORMULATION

In this section, we give the formulation of present study for a two-dimensional square lattice. The Hamiltonian of

the spin-3/2 BC model is given by:

$$\hat{H} = -J \sum_{\langle ij \rangle} S_i S_j + \sum_i \Delta_i S_i^2, \quad (1)$$

where the first term is a summation over the nearest neighbor spins with $S_i = \pm 3/2, \pm 1/2$ and Δ_i , in the second term, represents random crystal field which is distributed according to a given probability distribution function:

$$G(\Delta_i) = \frac{1}{2} (\delta(\Delta_i - \Delta(1 + \alpha)) + \delta(\Delta_i - \Delta(1 - \alpha))), \quad (2)$$

here $\alpha \geq 0$, and half of the lattice sites subject to a crystal field $\Delta(1 + \alpha)$ and the remaining lattice sites have a crystal field $\Delta(1 - \alpha)$. In order to examine the nonequilibrium properties of the considered magnetic system, first we have to focus on the equilibrium properties following the definition given in Ref. [44]. In this context, we have derived the Gibbs free energy ($G = U - TS - BNm$) by making use of MFA in the following form:

$$\begin{aligned} g(m, t, d, b) &= G/N = \frac{zm^2}{2} - bm \\ &- \frac{t}{2} [\log(2 \exp(-\frac{9\ell_1}{4}) \ell_3 + 2 \exp(-\frac{\ell_1}{4}) \ell_4)] \\ &- \frac{t}{2} [\log(2 \exp(-\frac{9\ell_2}{4}) \ell_3 + 2 \exp(-\frac{\ell_2}{4}) \ell_4)]. \end{aligned} \quad (3)$$

where z , $t = T/J$, $b = B/J$ and $d = \Delta/J$ are the coordination number, the reduced temperature, the reduced magnetic field and reduced crystal field, respectively. In addition, for the sake of simplicity, the coefficients $\ell_i (i = 1, \dots, 4)$ are defined as follows:

$$\begin{aligned} \ell_1 &= \frac{d(1+\alpha)}{t}, & \ell_3 &= \cosh\left(\frac{3zm}{2t}\right), \\ \ell_2 &= \frac{d(1-\alpha)}{t}, & \ell_4 &= \cosh\left(\frac{zm}{2t}\right). \end{aligned} \quad (4)$$

In the absence of the external applied field (b) and by benefiting from the minimization condition of the free energy at the equilibrium ($\frac{\partial g}{\partial m} = 0$), the mean field equation of state is found as follows:

$$m = \frac{3\ell_5 + e^{2\ell_1}\ell_6}{4\ell_3 + 4e^{2\ell_1}\ell_4} + \frac{3\ell_5 + e^{2\ell_2}\ell_6}{4\ell_3 + 4e^{\ell_2}\ell_4}. \quad (5)$$

here $\ell_5 = \sinh\left(\frac{3zm}{2t}\right)$ and $\ell_6 = \sinh\left(\frac{zm}{2t}\right)$. This equation may be solved without difficulty by an iterative procedure, i.e. Newton-Raphson method. Because the detailed equilibrium investigations of the considered system are analyzed in Ref. [44], we shall only give a brief summary here as follows: Equilibrium phase diagrams of the spin-3/2 BC model sensitively depend on the selected value of α , and the system exhibits four kinds of behavior: For $0 \leq \alpha < 1/4$, phase transition line between ferromagnetic $F_{3/2}$ and the ferromagnetic $F_{1/2}$ phases consists of only first order phase transition points and

also it is found that $\alpha = 1/4$ is independent of the coordination number. Two successive discontinuous phase transition lines between $F_{3/2}$ and F_1 phases and between F_1 and $F_{1/2}$ phases occur for the interval $1/4 \leq \alpha < 1$. For $\alpha = 1$, a discontinuous phase transition line exists separating $F_{3/2}$ and F_1 phases. On the other hand, two discontinuous phase transition lines emerge for $\alpha > 1$, the aforementioned transitions are between F_1 and $F_{3/2}$ phases, and between $F_{3/2}$ and F_1 phases.

A. Magnetic Gibbs energy production, kinetic equations and relaxation time

In order to understand the relaxation behavior of the spin-3/2 BC model with quenched random crystal field, one supposes a small uniform external magnetic field is applied along z axis. When a system is disturbed from its equilibrium state, quantities decay rapidly to their equilibrium values [45]. In that case, new energy form of the considered magnetic system in the neighborhood of equilibrium state can be expressed in the following:

$$g(m, t, d, b) = g^0(m_0, t, d, b_0) + \Delta g, \quad (6)$$

where $g^0(m_0, t, d, b_0)$ is equilibrium magnetic Gibbs energy in the absence of b , and Δg is the energy variation of the system and it is expressed as follows:

$$\begin{aligned} \Delta g &= \frac{1}{2}\Lambda_1(m - m_0)^2 + \Lambda_2(b - b_0) \\ &+ \frac{1}{2}\Lambda_3(b - b_0)^2 + \Lambda_4(m - m_0)(b - b_0), \end{aligned} \quad (7)$$

where the coefficients $\Lambda_i (i = 1, \dots, 4)$ can be easily calculated by using Taylor expansion and are given in Appendix. According to theory of irreversible thermodynamics, the generalized force (X_m), which conjugates the current (\dot{m}), is derived by differentiating Δg with respect to $m - m_0$:

$$X_m = \frac{\partial \Delta g}{\partial (m - m_0)} = \Lambda_1(m - m_0) + \Lambda_4(b - b_0). \quad (8)$$

The magnetic system will be relaxed back to its equilibrium state with a small deviation in the external magnetic field strength, and related process can be investigated by using linear response theory because the deviation of the applied external magnetic field is so small. The linear relation between the generalized flux and force can be written in terms of phenomenological coefficient as:

$$\dot{m} = \gamma X_m, \quad (9)$$

$$\dot{m} = \gamma \Lambda_1(m - m_0) + \gamma \Lambda_4(b - b_0), \quad (10)$$

where γ is a kinetic rate coefficient. By assuming a simple form for the solution $m - m_0 \simeq \exp(-t/\tau)$, the linearized

equation of motion has been solved and the relaxation time can be found in the following form:

$$\tau = -\frac{1}{\gamma\Lambda_1}, \quad (11)$$

By solving Eq. (11) numerically for a combination of Hamiltonian parameters we obtained the dynamic evolution of the relaxation time of the spin-3/2 BC model with quenched random crystal field.

B. Derivation of kinetic equations leading to complex magnetic susceptibility

When a ferromagnetic material is subject to a periodically varying time-dependent magnetic field at an angular frequency ω , all quantities will oscillate near the equilibrium state at this same angular frequency:

$$m - m_0 = m_1 \exp(i\omega t), \quad b - b_0 = b_1 \exp(i\omega t). \quad (12)$$

One can easily obtain the following form substituting Eq. (12) into the kinetic equation given by Eq. (10):

$$m_1(\gamma\Lambda_1 - i\omega) + \gamma\Lambda_4 b_1 = 0. \quad (13)$$

Solving Eq. (13) for m_1/b_1 gives:

$$\frac{m_1}{b_1} = \frac{\gamma\Lambda_4}{i\omega - \gamma\Lambda_1}. \quad (14)$$

Eq. (14) can be used to calculate the complex magnetic susceptibility of the considered system $\chi(\omega)$ and the spin-3/2 BC model induced magnetization (total induced magnetic moment per unit volume) is given by:

$$m - m_\infty = \text{Re}(m_1 \exp(i\omega t)), \quad (15)$$

where m_∞ is the magnetization induced by a time dependent oscillating magnetic field. Also, by benefiting from definition, the expression for $\chi(\omega)$ may be written:

$$m - m_\infty = \text{Re}(\chi(\omega)b_1 \exp(i\omega t)), \quad (16)$$

where $\chi(\omega) = \chi'(\omega) - i\chi''(\omega)$ is the complex magnetic susceptibility and its real and imaginary parts are called as magnetic dispersion and absorption factors, respectively. Finally the magnetic dispersion and absorption factors can be found in the following forms:

$$\begin{aligned} \chi'(\omega) &= \frac{\gamma\Lambda_4\tau}{1 + \omega^2\tau^2}, \\ \chi''(\omega) &= \frac{\gamma\Lambda_4\tau^2\omega}{1 + \omega^2\tau^2}. \end{aligned} \quad (17)$$

III. RESULTS AND DISCUSSION

In this section, we will focus our attention on the nonequilibrium dynamics properties of the spin-3/2 BC model with quenched crystal field. This section is divided into two parts as follows: In section III A, we discuss how the quenched random crystal field affect the relaxation time behavior in the vicinity of order-disorder transition temperature as well as near the isolated critical point. In addition, we examine the crystal field variation of the relaxation time for selected Hamiltonian parameters, and also we calculate dynamical mean field critical exponents in order to formulate the critical behavior of relaxation time of the considered system. Real and imaginary parts of the complex magnetic susceptibility are analyzed near the second-order phase transition temperature and also near the isolated critical point in section III B. As a final investigation, dynamical mean field critical exponents are calculated for magnetic dispersion and absorption factors for both low and high frequency regions.

A. Relaxation time treatment of the spin-3/2 BC model with quenched crystal field

Fig. 1(a) represents the phase diagram in the reduced crystal field and reduced temperature (d, t) plane for $\alpha = 0.05$. We should note that this type of phase diagram can be observed by selecting any value in the range of $0 \leq \alpha < 1/4$. Here, the full and dotted lines denote the continuous and discontinuous phase transition points, respectively, and also the solid circle indicates the isolated critical point. One can clearly see from the Fig. 1(a) that the ferromagnetic phase separates the paramagnetic phase with a phase boundary line containing continuous phase transition points, and also phase transition line between ferromagnetic $F_{3/2}$ and the ferromagnetic $F_{1/2}$ phases consists of only first order phase transition points at the relatively lower temperatures. Fig. 1(b) shows the thermal variation of the relaxation time τ in the neighborhood of order-disorder phase transition temperatures for selected values of d , γ and α . In this figure, the solid and dashed dotted curves correspond to values of $d = -3.0$ and -4.0 for $\alpha = 0.05$ and $\gamma = -0.1$, respectively. The vertical dashed lines refer to the phase transition temperatures for each value of the reduced crystal field. As seen in Fig. 1(b) τ grows rapidly with increasing temperature and diverges as the temperature approaches the order-disorder phase-transition points for both crystal field values $d = -4.0$ and -3.0 . In order to understand the influences of the Onsager rate coefficient on the relaxation time, we give the Fig. 1(c) for varying γ values, and selected values of $\alpha = 0.05$ and $d = 4.0$. On both sides of the critical temperature, illustrated by a vertical dashed line, the relaxation time is divergent for all γ values. It is worthwhile to stress that even though the divergence of τ gets pushed away from the critical temperature when the absolute value of the On-

sager rate coefficient decreases, there exists a qualitative consistency between their thermal variations. Aforementioned behaviors are in accordance with the previously published studies [19–24]. On the other hand, Fig. 1(d) corresponds to crystal field dependence of τ near the first order phase transition point between ferromagnetic $F_{3/2}$ and $F_{1/2}$ phases for values of $\alpha = 0.05$, $t = 0.6$ and $\gamma = -0.1$. In contrary to treatment of the second order phase transition, τ exhibits a finite discontinuity at the first order phase transition point.

In the following analysis, let us focus on the influences of α parameter on the relaxation dynamics of the considered magnetic system. In Fig. 2(a), we plot the phase diagram in (d, t) plane with selected value of $\alpha = 0.3$. It can be easily seen from the Fig. 2(a) that there exists two successive first order phase transitions at the relatively low temperature. The first transition is from ferromagnetic $F_{3/2}$ phase to ferromagnetic F_1 phase while the second transition is between ferromagnetic F_1 and $F_{1/2}$ phases. As mentioned in section II, similar type of phase diagrams can be obtained by focusing the range of $1/4 \leq \alpha < 1$. On the other side, Fig. 2(b) demonstrates that the τ exhibits two successive infinite discontinuous for values of $\alpha = 0.3$, $t = 0.47$ and $\gamma = -0.1$ in which the first one is between ferromagnetic $F_{3/2}$ and ferromagnetic F_1 phases and second one is between ferromagnetic F_1 and ferromagnetic $F_{1/2}$.

In Fig. 3(a), for a bigger α parameter, for example $\alpha = 1.0$, a discontinuous phase transition line exists separating $F_{3/2}$ and F_1 phases. The crystal field variation of the τ is shown in Fig. 3(b) corresponding to the same α parameter for values of $t = 0.4$ and $\gamma = -0.1$. It can be clearly seen from the Fig. 3(b) that as the crystal field increases, τ makes a sharp cusp at the critical crystal field value. As another characteristic feature of the considered system we have examined the crystal field variation of τ in the neighborhood of isolated critical point. As it was expected, in Fig. 3(c), we found that τ shows a divergence treatment for selected values of parameters such as $\alpha = 1.0$, $t = 0.51$ and $\gamma = -0.1$.

The divergence of the relaxation time on both sides of the critical point can be described by the critical exponents. Thus, we may assume that, for temperatures lower than the transition temperatures, τ obeys the law of the form $\tau(t) \approx (t_c - t)^\lambda$ and values of the λ can be obtained from the plot of $\log(\tau) - \log(\epsilon)$, where $\epsilon = \frac{t_c - t}{t_c}$ [45]. In Fig. 4(a), we have plotted τ versus dimensionless parameter (ϵ) in the neighborhood of continuous phase transition point, and we have obtained only one linear part on this plot for τ . It is found that the slope of the curve equals to -1.004 ± 0.001 indicating that τ diverges with a mean field exponent of -1.004 ± 0.001 as $(t_c - t)^{-1.004 \pm 0.001}$ for the selected values of parameters. In addition to these, a similar analyze has been done for isolated critical point in Fig. 4(b). It is deduced from the log-log plot of τ versus dimensionless parameter that the slope of the curve equals to -0.997 ± 0.001 which indicates that the τ of the spin-3/2 BC model with quenched ran-

dom crystal field diverges with a mean-field exponent of -0.997 ± 0.001 as $(t_c - t)^{-0.997 \pm 0.001}$ for values of $d = 1.25$, $\alpha = 1.0$ and $\gamma = -1.0$.

It can be seen clearly from the Fig. 5(a) that the magnetic system exhibits reentrant behavior at the relatively higher temperature regions for $\alpha = 2.5$. This behavior originates from the competition between negative and positive values of crystal field interactions. And also, two discontinuous phase transition lines occur at the low temperature regions. The first one is between the ferromagnetic F_1 and ferromagnetic $F_{3/2}$ phases, and the second one is from the ferromagnetic $F_{3/2}$ to F_1 phases. In order to show the reentrant behavior on the relaxation times, we give the Fig. 5(b) for varying values of γ and selected values of t , and d . As was expected, two successive divergences indicating two successive second order phase transition points exist in Fig. 5(b) for values of $\alpha = 2.5$, $t = 4.5$ where the first one is from ordered phase to disordered phase while the other is between the disordered and ordered phases, respectively. It is necessary to say that as the absolute value of the Onsager rate coefficient decreases (γ), divergence of τ gets pushed away from the critical temperature. As a final investigation, we focused on the low temperature region and we gave the crystal field variation of τ in Fig. 5(c). One can see from the figure Fig. 5(d), τ shows two infinite discontinuous corresponding to phase transition points, as crystal field interaction increases for selected values such as $\alpha = 2.5$, $t = 0.4$ and $\gamma = -0.1$.

B. Complex magnetic susceptibility treatment of the spin-3/2 BC model with quenched crystal field

Figs. 6(a-d) illustrate the real and imaginary parts of the ac susceptibility in the vicinity of isolated critical point for both low and high frequency regions and values of $d = 1.25$, $\alpha = 1.0$ and $\gamma = -0.1$. In Figs. 6(a-b) the temperature variations of the magnetic dispersion factors are given, and, one can easily see from the figures that the χ' increases rapidly with increasing temperature and exhibits a divergence near the isolated critical point for the low frequency values. On the other hand, the χ' shows a shallow dip at the isolated critical point for the relatively higher applied field frequencies. In addition, two frequency dependent local maxima occur at before and after the isolated critical point, respectively (see Fig. 6(b)). From the Figs. 6(c-d) we see that the χ'' exhibits a divergence at the isolated critical point for the low frequency regions, while it displays a local maxima for the relatively higher applied field frequencies. By comparing figures 6(a) and (c) one can see that the divergence treatment of χ' is independent of the applied field frequency whereas divergence of χ'' sensitively depends on the frequency. On the other hand, it is possible to say that the similar ac magnetic susceptibility behavior can be observed by focusing any second order phase transition point.

In the following analysis, let us investigate the reentrant behavior properties on the magnetic dispersion and absorption factors corresponding to the phase diagram in Fig. 5(a). In Fig. 7(a) we represent the crystal field variation of the magnetic absorption factor for selected values of t, γ and α with different applied field frequencies in the low frequency region. One can observe from the figure that the magnetic system exhibits two successive divergences behavior indicating two successive second order phase transition points where the first one is from ordered phase to disordered phase while the other is between the disordered and ordered phases, respectively. On the other hand, as seen from figure 7(b), the magnetic absorption factor has two frequency dependent shallow dips at the second order phase transition points in the high frequency region. In addition, it is useful to mention that as we increase the applied field frequency, the maximum located at a finite crystal field value in the ordered and disordered phases decreases.

As a final investigation, dynamical mean field critical exponents for isolated critical point are calculated for magnetic dispersion and absorption factors for both low and high frequency regions in Fig. 8. It is known that the critical behaviors of χ' and χ'' exponents in the neighborhood of critical point as well as isolated critical point are characterized by the critical exponents. Thus, we may assume that, for example, when $t \rightarrow t_{ic}$, χ' and χ'' follow scaling laws of the form $\chi' \sim |t - t_{ic}|^{\lambda'}$ and $\chi'' \sim |t - t_{ic}|^{\lambda''}$, respectively. Here, λ', λ'' and t_{ic} are dynamical mean field critical exponents of the magnetic dispersion and absorption factors and isolated critical point of the considered system, respectively. In order to calculate the value of the dynamical critical exponents one should sketch double logarithmic plot of the related quantity versus $\log(1 - t/t_{ic})$ and find the slope of the linear part of this curve. This definition is valid for all values of the exponents where the negative values correspond to the divergence of the related quantity as $1 - t/t_{ic}$ goes to zero and the positive values correspond to logarithmic divergence, cusps or jump singularities [19, 45]. Fig. 8(a) represents the plots of $\log(\chi')$ and $\log(\chi'')$ versus $\log(1 - t/t_{ic})$ at several values of applied field frequency corresponding to high region in the vicinity of isolated critical point for values of $\alpha = 1.0$, $d = 1.25$ and $\gamma = -0.1$. We should mention here that we have found only one linear part on each of these log - log plots, namely, one value for λ' , and one value for λ'' , respectively. It is clearly seen from the figure that the behavior of $\log(\chi')$ and $\log(\chi'')$ are dependent the applied field frequency, and the slopes of the $\log(\chi')$ and $\log(\chi'')$ are equal to 1.0 and 0, respectively. Namely, although the behaviors of the magnetic dispersion and absorption factors depend on the field frequency in the high frequency region, their slopes are not dependent the ω . This scaling form ($\lambda' = 1.0$) verifies the convergence of the χ' to zero in the high-frequency region in the neighborhood of isolated critical point (see Fig. 6(b)). In addition, scaling form ($\lambda'' = 0.0$) of the χ'' implies the existence a local

maxima in the high frequency region, in other words, it is in accordance with Fig. 6 (d). On the other hand, in Fig. 8 (b), we depict the plots of $\log(\chi')$ and $\log(\chi'')$ versus $\log(1 - t/t_{ic})$ at several values of applied field frequency corresponding to low region in the vicinity of isolated critical point for the same values with Fig. 8 (a). One can figure out from Fig.8 (b) that the logarithmic behavior of magnetic dispersion factor does not depend on the applied field frequency whereas the magnetic absorption factor changes with ω in the low frequency region. The mean field dynamic critical exponent of the $\log(\chi')$ and $\log(\chi'')$ are equal to -1 and -2 , respectively. It is obvious that these scaling forms ($\lambda' = -1.0, \lambda'' = -2.0$) correspond to divergence behavior in the vicinity of isolated critical point in the low frequency region and for selected system parameters.

IV. CONCLUDING REMARKS

In this study, we have studied the nonequilibrium dynamics of the spin-3/2 BC model with quenched random crystal field by means of Onsager irreversible thermodynamics. In order to obtain the Gibbs free energy of the considered disordered system, we used the MFA, and by benefiting from the minimization condition of Gibbs free energy, we found the mean field equation of state and we represented the phase diagram in the (d, t) plane for different α parameters. After that, we have calculated the Gibbs free-energy production in the irreversible process and by means of this generalized force and flux are defined in irreversible thermodynamics limit. Then, the kinetic equation for the magnetization is obtained by using linear response theory. From the solution of the kinetic equation in the vicinity of equilibrium states, a relaxation time describing the nonequilibrium dynamics of the considered system is obtained. In order to understand the critical behavior of the relaxation phenomena, the temperature and also crystal field dependencies of the relaxation time are determined in the vicinity of the phase-transition points. We found that τ grows rapidly with increasing temperature and diverges as the temperature approaches the order-disorder phase-transition points as well as in the vicinity of isolated critical point for selected Hamiltonian parameters and Onsager rate coefficients. And also we calculated the dynamical mean field critical exponents for order-disorder phase transition and isolated critical points, respectively and it is found that the slopes of the curves are equal to -1.004 ± 0.001 and -0.997 ± 0.001 indicating that τ diverges with increasing temperature. In contrary to treatment of the second order phase transition, τ exhibits a finite discontinuity at the first order phase transition point for fixed temperature value. In addition, as mentioned above we have touched upon the influences of Onsager rate coefficient on the τ behavior, and it is observed that even though divergence of τ gets pushed away from the critical temperature when the absolute value of the Onsager

rate coefficient decreases (γ), there exists a qualitative consistency between their thermal variations. Aforementioned behaviors are in accordance with the previously published studies [19–24].

Moreover, we have analyzed the complex magnetic susceptibility treatment of the spin-3/2 BC model with quenched random crystal field under the time dependent oscillating magnetic field. It is worthwhile to mention that the amplitude of applied magnetic field is so small that we made use of linear response theory in studying the magnetic relaxation process. We especially have touched upon the thermal variation of complex magnetic susceptibility near the isolated critical point for varying applied field frequencies. We found that the magnetic dispersion and absorption factors grow drastically and diverges with increasing temperature in the low frequency regions (see Figs. 6(a) and (c)). In addition, we can say that the divergence treatment of χ' is independent of the field frequency whereas divergence of χ'' sensitively depends on the frequency. On the other hand, the χ' and χ'' show a shallow dip and a local maxima at the isolated critical point for the relatively higher applied field frequencies, respectively. It is also observed that two frequency dependent local maxima occur at before and after the isolated critical point, respectively. Moreover, we have investigated the crystal field dependence of the complex magnetic susceptibility in the neighborhood of reentrant region for selected Hamiltonian parameters and we found that the magnetic system exhibits two successive divergences behavior indicating two successive second order phase transition points in the low frequency region. For higher frequency region, the magnetic absorption factor has two frequency dependent shallow dips at the second order phase transition points. In addition, it is useful to mention that as we increase the applied field frequency, the maximum located at a finite crystal field value in the ordered and disordered phases decreases. As a final investigation we have calculated the dynamical mean field critical exponents of complex magnetic susceptibility for isolated critical point. We also mention that a qualitatively good agreement exists between our findings and previously published studies [18, 19, 40].

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V. APPENDIX

The coefficients $\Lambda_i (i = 1, \dots, 4)$ can be easily calculated by using Taylor expansion and are given as follows:

$$\begin{aligned}\Lambda_1 &= \left(\frac{\partial^2 g}{\partial m^2} \right)_{eq} = \\ & z + \frac{t}{2} \left(\frac{\frac{3 \exp\left(\frac{-9\ell_1}{4}\right)\ell_5 z + \exp\left(\frac{-\ell_1}{4}\right)\ell_6 z}{t}}{2 \exp\left(\frac{-9\ell_1}{4}\right)\ell_3 + 2 \exp\left(\frac{-\ell_1}{4}\right)\ell_4} \right)^2 \\ & + \frac{t}{2} \left(\frac{\frac{3 \exp\left(\frac{-9\ell_2}{4}\right)\ell_5 z + \exp\left(\frac{-\ell_2}{4}\right)\ell_6 z}{t}}{2 \exp\left(\frac{-9\ell_2}{4}\right)\ell_3 + 2 \exp\left(\frac{-\ell_2}{4}\right)\ell_4} \right)^2 \\ & - \frac{t}{2} \left(\frac{\frac{9 \exp\left(\frac{-9\ell_1}{4}\right)\ell_3 z^2 + \exp\left(\frac{-\ell_1}{4}\right)\ell_4 z^2}{2t^2}}{2 \exp\left(\frac{-9\ell_1}{4}\right)\ell_3 + 2 \exp\left(\frac{-\ell_1}{4}\right)\ell_4} \right) \\ & - \frac{t}{2} \left(\frac{\frac{9 \exp\left(\frac{-9\ell_2}{4}\right)\ell_3 z^2 + \exp\left(\frac{-\ell_2}{4}\right)\ell_4 z^2}{2t^2}}{2 \exp\left(\frac{-9\ell_2}{4}\right)\ell_3 + 2 \exp\left(\frac{-\ell_2}{4}\right)\ell_4} \right), \\ \Lambda_2 &= \left(\frac{\partial g}{\partial b} \right)_{eq} = -m, \\ \Lambda_3 &= \left(\frac{\partial^2 g}{\partial b^2} \right)_{eq} = 0, \\ \Lambda_4 &= \left(\frac{\partial^2 g}{\partial m \partial b} \right)_{eq} = -1.\end{aligned}$$

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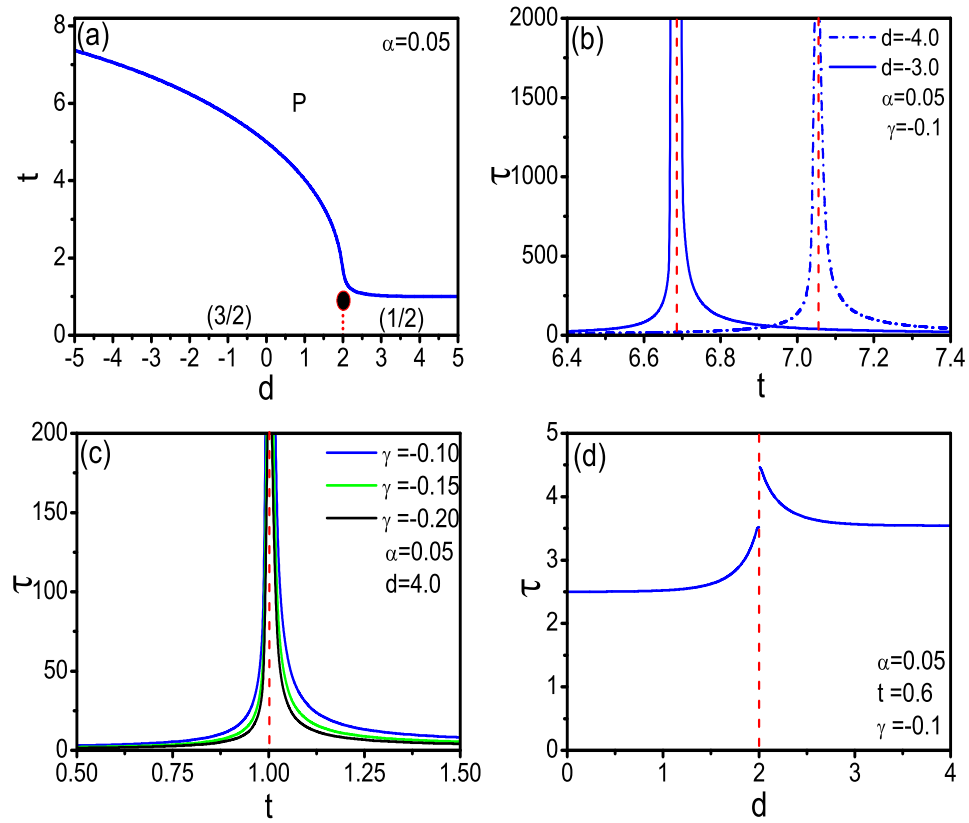


FIG. 1: (a) Phase diagram of the spin-3/2 BC model with quenched random crystal field for $\alpha = 0.05$. The full and dotted lines denote the continuous and discontinuous phase transitions, respectively. The solid circle represents the isolated critical point. (b) Thermal variation of the relaxation time for different values of the crystal fields near the critical point. (c) Effects of the γ parameters on the relaxation time in the neighborhood of critical point. (d) Crystal field variation of the relaxation time in the neighborhood of discontinuous phase transition.

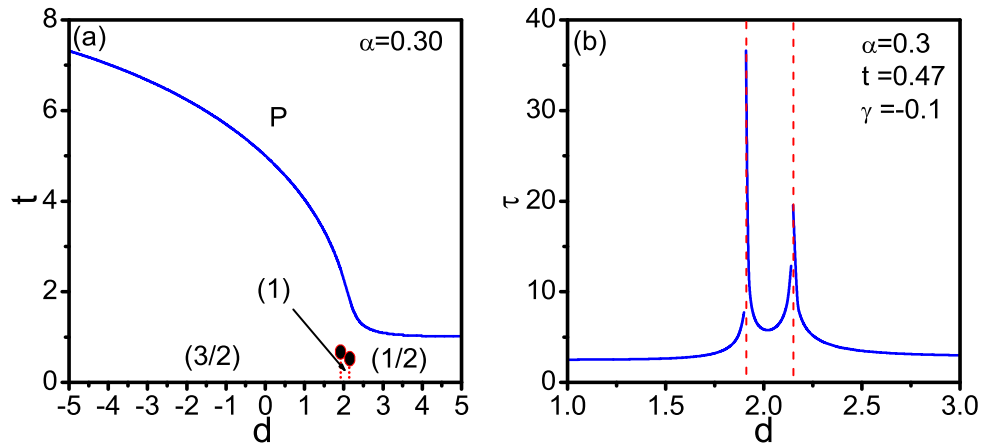


FIG. 2: (a) Phase diagram of the spin-3/2 BC model with quenched random crystal field for $\alpha = 0.30$. The full and dotted lines denote the continuous and discontinuous phase transition points respectively. The solid circles represent the isolated critical points. (b) Crystal field variation of the relaxation time in the neighborhood of discontinuous phase transitions.

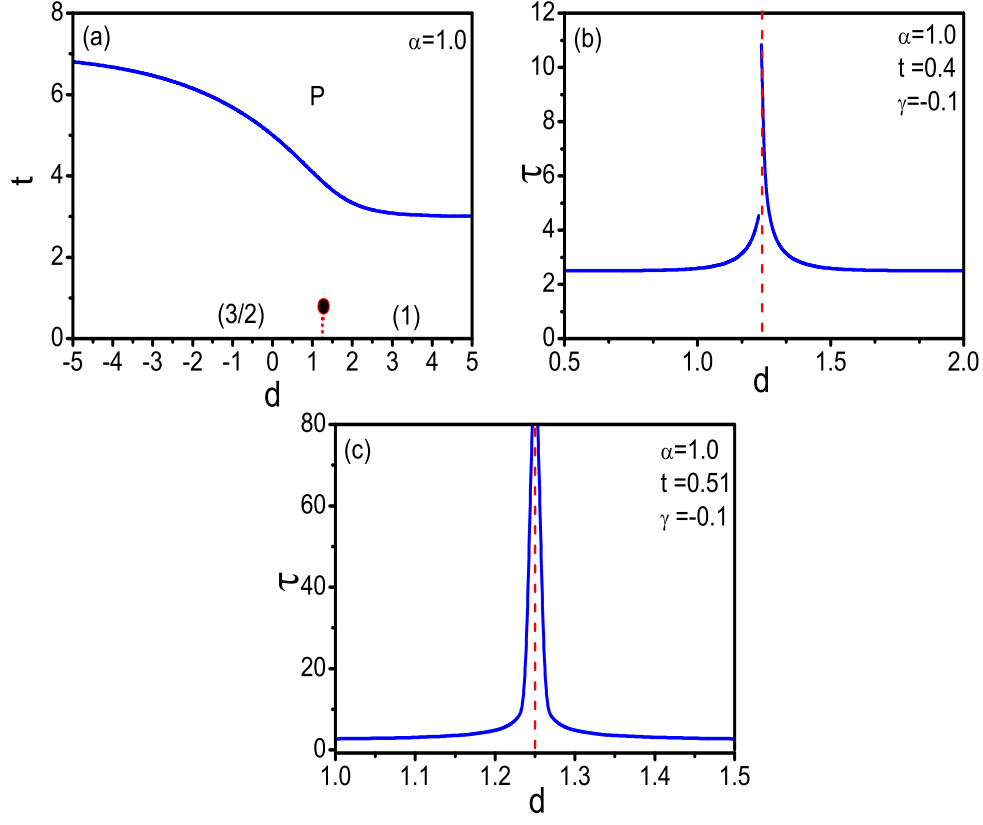


FIG. 3: (a) Phase diagram of the spin-3/2 BC model with quenched random crystal field for $\alpha = 1.0$. The full and dotted lines denote the continuous and discontinuous phase transition points respectively. The solid circle represents the isolated critical point. Crystal field variations of the relaxation time in the vicinity of discontinuous (b) and isolated critical point (c).

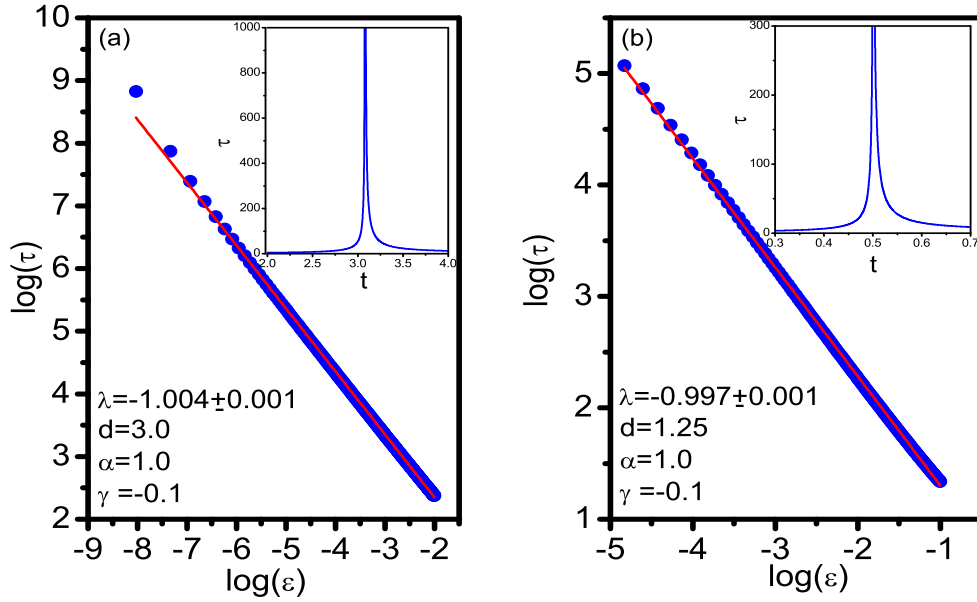


FIG. 4: Log-log plot of relaxation time vs. reduced parameter, and in the inset, relaxation time vs. temperature are shown near the continuous phase transition point (a) and near the isolated critical point (b).

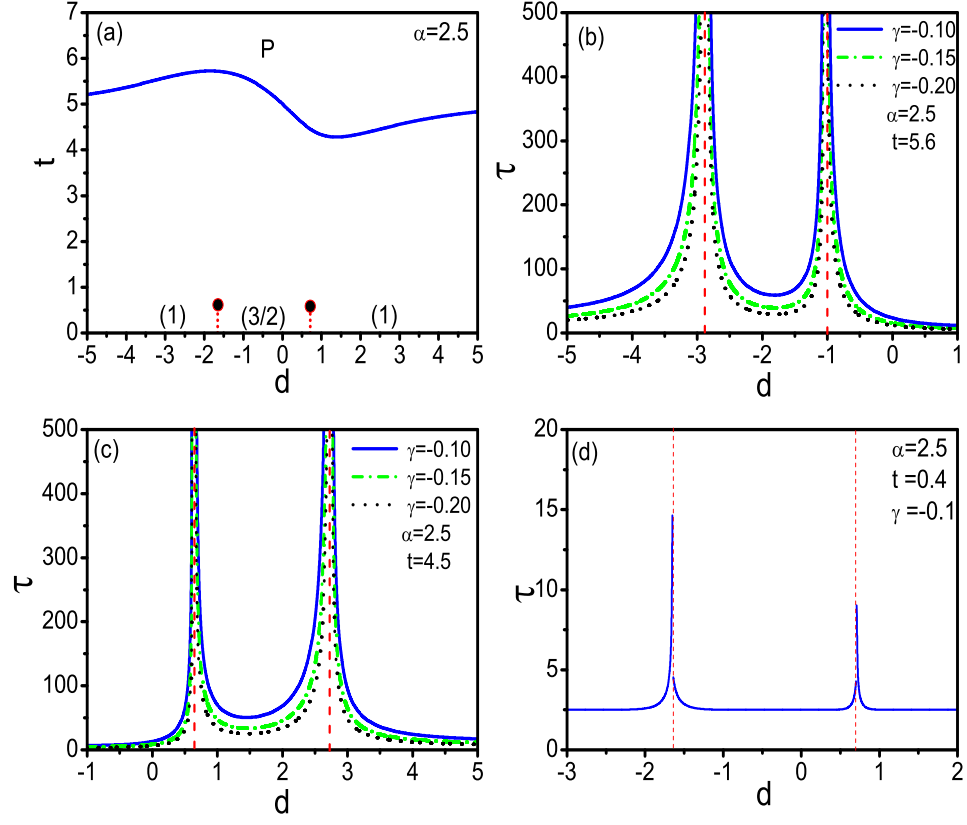


FIG. 5: (a) Phase diagram of the spin-3/2 BC model with quenched random crystal field for $\alpha = 2.5$. The full and dotted lines denote the continuous and discontinuous phase transition points, respectively. The solid circles represent the isolated critical points. (b) Effect of the kinetic rate coefficient on the reentrant behavior at $t = 5.6$ (c) Effect of the kinetic rate coefficient on the reentrant behavior at $t = 4.5$ (d) Crystal field variation of the relaxation time in the neighborhood discontinuous phase transitions.

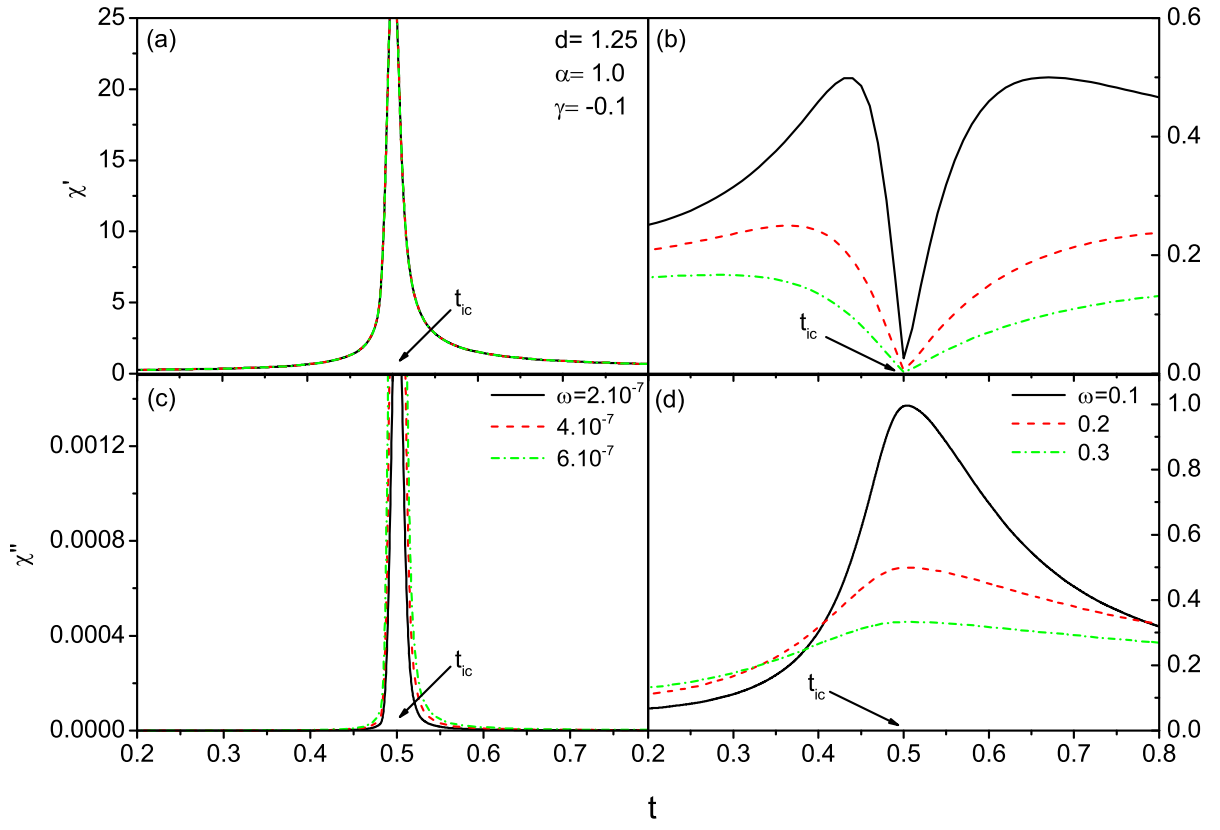


FIG. 6: Temperature variations of magnetic dispersion and absorption factors for the low (a and c) and high frequency (b and d) region at several values of the frequency (ω) in the vicinity of isolated critical point while $d = 1.25$, $\alpha = 1.0$ and $\gamma = -0.1$. The arrow refers to the isolated critical point.

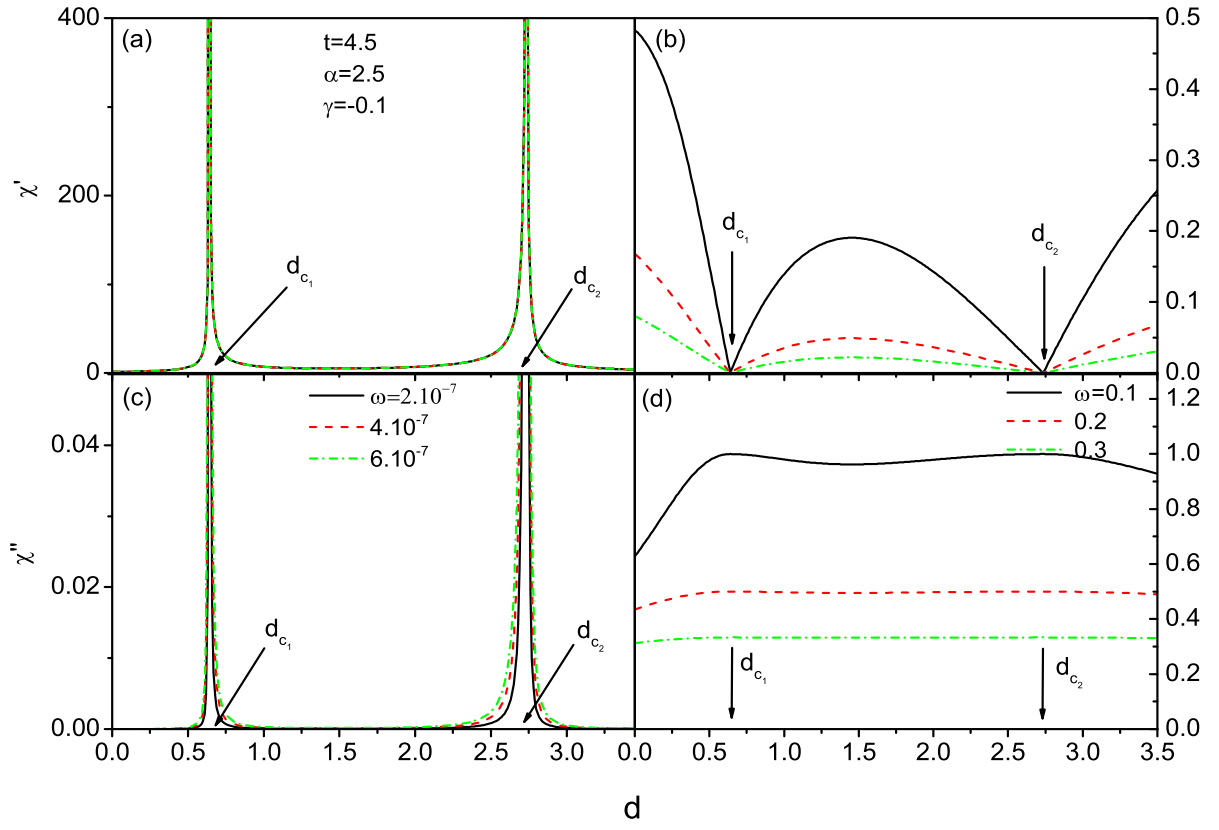


FIG. 7: Crystal field variations of magnetic dispersion and absorption factors for the low (a and c) and high frequency (b and d) regions at several values of the frequency (ω) in the vicinity of reentrant region while $t = 4.5$, $\alpha = 2.5$ and $\gamma = -0.1$. The arrows refer to the critical crystal field values.

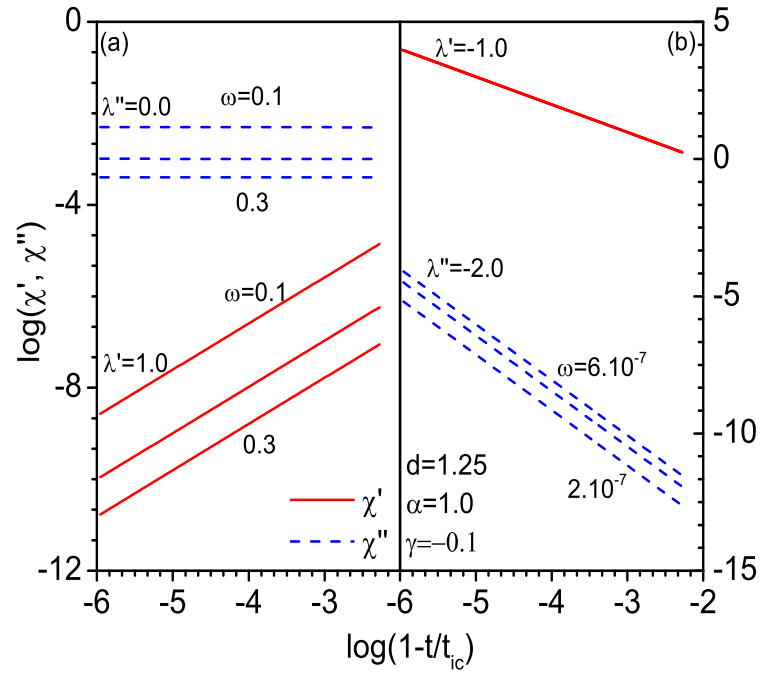


FIG. 8: Double logarithmic (log-log) plots of magnetic dispersion and absorption factors versus $(1 - t/t_{ic})$ at several values in the high (a) and low (b) frequency regions while $d = 1.25$, $\alpha = 1.0$ and $\gamma = -0.1$. Solid and dashed lines denote the $\log(\chi')$ and $\log(\chi'')$, respectively.